1. The half-life of a certain cobalt isotope is 5 years, i.e., in any sample it takes 5 years for half of the atoms to decay. After a nuclear accident the concentration of this isotope found at the site is 7 times the maximum level considered acceptable for human habitation. The law governing radioactive decay says that, if A is the total amount of the isotope at the site t years after the accident, then

$$dA/dt = -kA$$

where k is a constant > 0.
a) Find k.
b) Find the number of years it will take for the site to be fit for human occupation.

2. A ladder 5 meters long is leaning against a high vertical wall when its base begins to slip horizontally away from the wall. The distance s from the base to the wall (measured in meters) satisfies the differential equation

$$ds/dt = 1 + e^{-1}.$$

(Time is measured in seconds.)
Consider the area of the right-angled triangle formed by the ladder, the wall, and the ground. At the instant when the top of the ladder is 3 metres above the ground,
a) is this area increasing or decreasing?
b) what is the area's exact rate of change? (Give units with your answer.)

3. Let $f(x) = e^{x-2} + x^3 - 2$
a) Use the derivative of f to explain why the equation $f(x) = 0$ has at most one solution.
b) Explain why $f(x) = 0$ has a solution in the interval $(1; 2)$.

4. For each positive number a, let $V(a)$ denote the maximum value of $(a/x)^x$ on the interval where $x > 0$.
i) Find an explicit formula for $V(a)$.
ii) Prove that $V'(a)/V(a)$ is independent of a and find its value.

5 Let $f : [0, +\infty) \to [0, +\infty)$ be a differentiable map satisfies the equation

$$f'(x) \leq kf(x), f(0) = 0$$

where k is a constant. Show that $f \equiv 0$.